**Seminar SAW : History of Mathematics, History of Economical and Financial Practices**
"Constructions and excavations", Paris, 6 April 2012

# Mathematics and the planning of public works in China, Han to Yuan

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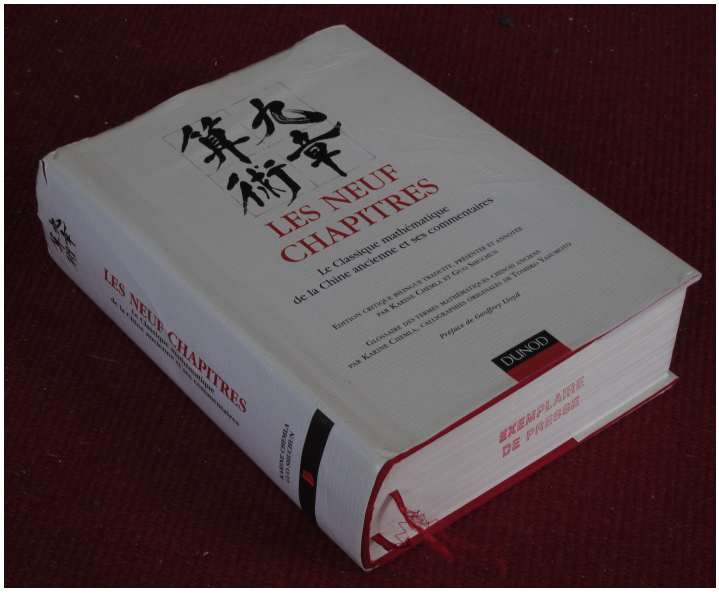
www.donwagner.dk/saw.pdf




***JIUZHANG SUANSHU* 九章算術**

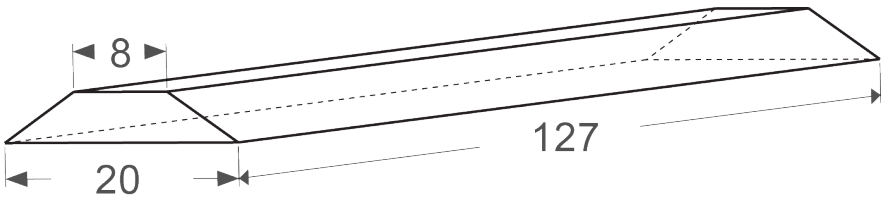
**ARITHMETIC IN NINE CHAPTERS**
**1ST CENTURY CE?**

# *Jiuzhang suanshu*, ch. 5, problem 4

A dyke has breadth 2 *zhang*, upper breadth 8 *chi*, height 4 *chi*, and length 12 *zhang* 7 *chi*.

What is the volume?

Answer: 7112 [cubic] *chi*. $\frac{(20+8)\times 4\times 127}{2} = 7112\ chi^3$




# JZSS, 5: 4, continued

By the winter norm, one person's labour is 444 [cubic] *chi*.

How many labourers are used?

Answer: $16\ {}^{2}/_{111}$

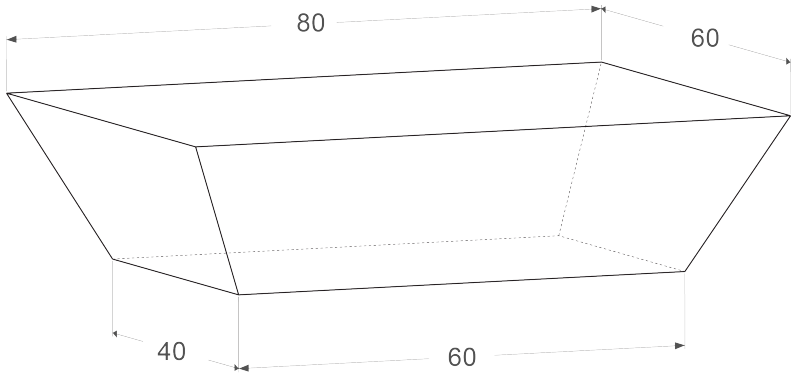
Method: . . .

$$\frac{7112\ chi^3}{444\ chi^3 / \text{labourer}} = 16\ {}^{2}/_{111}\ \text{labourers}$$

# JZSS, 5: 21

A pool has upper breadth 6 *zhang*, length 8 *zhang*, lower breadth 4 *zhang*, length 6 *zhang*, and depth 2 *zhang*. What is the volume?

Answer: 70,666 $^{2}/_{3}$ [cubic] *chi*.




# JZSS 5: 21, continued

The carrying of the earth is 70 paces, with 20 paces up and down wooden steps. Two [paces] on the steps correspond to five on a level path. For resting time, one is added for each ten. Time for loading and unloading is equivalent to 30 paces. One round trip is determined to be 140 paces. The capacity of a basket of earth is 1 *chi* 6 *cun* [i.e. 1.6 cubic *chi*]. The autumn norm for one person's labour is equivalent to walking 59½ *li* [ca. 22 km].

What is the volume [of earth] carried by each person, and how many labourers are used?

Answer: One person carries 240 [cubic] *chi*, and the number of labourers is 346 $^{62}/_{153}$.

Method: • • •

# Digging a canal, ca. 1840

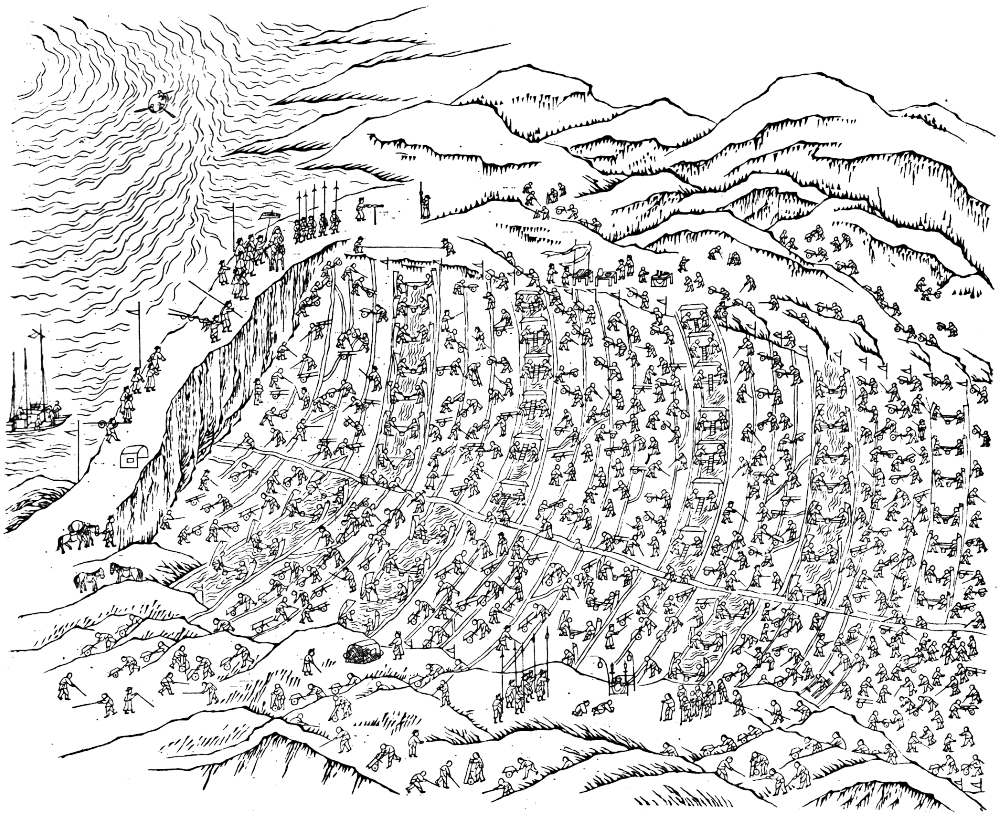


Fig. 876. The Chinese genius for organising very large numbers of workers in civil engineering operations illustrated by a unique drawing from the autobiography of the high official and hydraulic engineer Lin-Chhing (*Hung Hsüeh Yin Yuan Thu Chi*, ch. 3, pp. 81*a*, *b*, 82*a*).

# Building the Red Flag Canal, 1958–65

http://www.youtube.com/watch?v=wtkiJk9nGBM

# XU SHANG 許商, 1ST CENTURY BCE

## Xu Shang 許商, 1st century BCE

- Held numerous high positions in government (*Han shu*, 19b: 836, 841–2).
- Wrote a book in one chapter on natural phenomena, *Wu xing zhuanji* 五行傳記 (*Han shu*, 30: 1705). Possibly concerned with astronomy or the calendar (*Han shu buzhu*, 88: 13a).
- Wrote a book in 26 chapters on calendrical and astronomical calculations, *Xu Shang suanshu* 許商算術 (*Han shu*, 30: 1766).

(See also Loewe, *Biographical dictionary*, p. 622.)

# Xu Shang's story – background

- (About 100 BCE?) the Yellow River had bifurcated, forming a tributary which was called the Tunshi River 屯氏河.
- In 39 BCE the Tunshi River silted up.
- In 32 BCE Feng Qun 馮逡 reported that the Yellow River was in danger of overflowing its dykes, and that dredging the Tunshi River would ease the pressure and reduce the danger of flooding.

*Han shu*, 9: 1686–1699 (See also Needham, *Science and civilisation in China*, 4.3: 329–331).

# Xu Shang gives advice –

"The Emperor passed the memorial to the Chancellor and the Imperial Counsellor. They responded that Xu Shang was an authority on the Book of Documents, that he was good at calculating, and that he could estimate the labour required. He was sent to inspect the situation. He reported that the Tunshi River was the cause of flooding, but that the local labour resources were not sufficient, and dredging could be postponed."

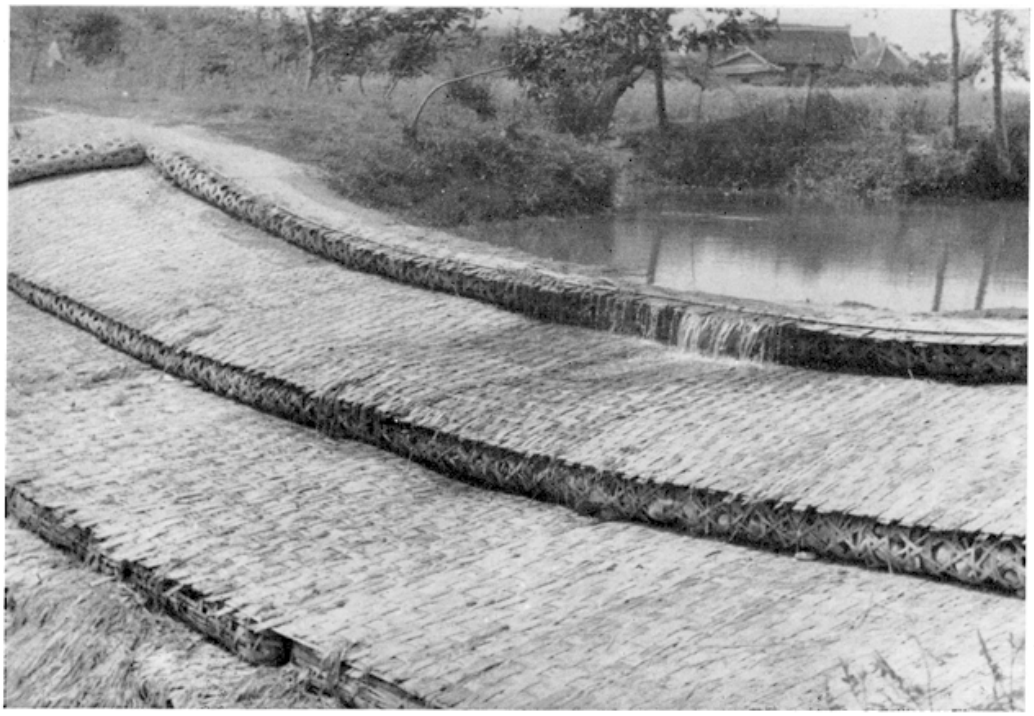
# – unfortunate advice

- Three years later the Yellow River overflowed its dykes and caused devastation in four commanderies and 32 prefectures.
- Wang Yanshi 王延世 was sent, and he repaired the dyke with stone-filled gabions.

**THE YELLOW RIVER**

# The Yellow River Delta


# Gabions



Needham, *Science and civilisation in China*, 4.3, pl. 381.

Fig. 913. A weir of gabions near Chhêngtu in Szechuan (orig. photo., 1943). The plaited bamboo cylinders filled with stones can be built into any desired formation; in structures such as this the successive layers are covered with bamboo matting. Cf. pp. 295, 321 ff., 339 ff. and Figs. 895, 914.

# More gabions

O. J. Todd, *Geographical review*, 1949, 39.1: 43.

FIG. 8—Closing the final gap in the 5000-foot-wide break at Huayuankou by use of long willow-stone "sausages."

## XU SHANG 許商, CONTINUED

# Problems with the experts

- Two years later the Yellow River again overflowed its dykes. It was proposed that Wang Yanshi should again be sent, but Du Qin 杜欽 objected that another expert, Yang Yan 楊焉, should be sent, together with Xu Shang and Shengma Yannian 乘馬延年.

"Wang Yanshi and Yang Yan will certainly have violent disputes; there will be deep discussions and mutual criticism. Xu Shang and Shengma Yannian both understand calculation and can estimate labour requirements, so that they can distinguish truth from error. They can choose the correct plan and follow it, so that there surely will be success."

- It was a success, and Wang Yannian was rewarded with 600 *jin* 斤 of gold (ca. 150 kg).

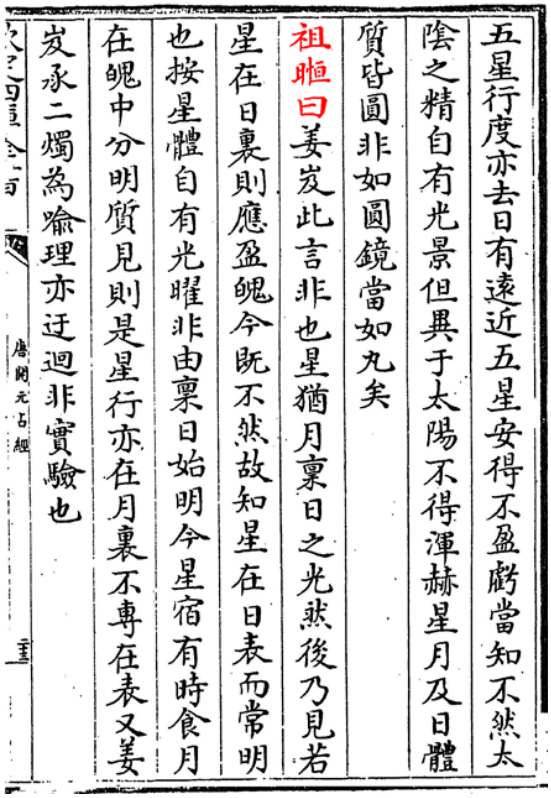
## ZU GENG 祖暅, 5TH CENTURY CE

# Zu Geng 祖暅, 5th century CE

- Son of the mathematician Zu Chongzhi 祖冲之.
- Used a version of Cavalieri's Theorem.
- Derived the correct formula for the volume of a sphere.
- Wrote something about astronomy.

# Zu Geng quoted in *Kaiyuan zhanjing* 開源占經

"This statement by Jiang Ji is wrong: 'Stars are like the moon, receiving [light] from the sun and only then being visible.' If stars were inside the sun [?], they would necessarily have phases; this is not the case. It is well known that stars • • • have a constant brightness. Thus the bodies of stars have their own light and do not receive [light] from the sun before they shine. • • •"


五星行度亦去日有遠近五星安得不盈虧當知不然太陰之精自有光景但異于太陽不得渾赫星月及日體質皆圓非如圓鏡當如丸矣

祖暅曰姜岌此言非也星猶月稟日之光然後乃見若星在日裏則應盈魄今既不然故知星在日表而常明也按星體自有光曜非由稟日始明今星宿有時食月在魄中分明質見則是星行亦在月裏不專在表又姜岌承二燭為喻理亦迂迴非實驗也

# Zu Geng gives unwanted advice ...

(*Liang shu* 梁書, 20:291–2; *Nan shi* 南史, 45: 1374–5)

- About 500 CE, Wang Zu 王足 presented calculations and proposed that the Huai River 淮水 should be dammed to provide irrigation for Shouyang 壽陽.

"The Emperor Gaozu 高祖 was in favour of this, and sent the waterworks master Chen Chengbo 陳承伯 and the Construction Supervisor [*caiguan jiangjun* 材官將軍] Zu Geng to inspect the topography. They both reported that the sandy mud of the Huai River was too light and insubstantial, so that the project could not be completed."

- The Emperor did not accept this, and set some 200,000 labourers to work building the dam.

# ... and ends up in prison.

- The dam was to be closed in 515, but various difficulties were encountered, the people were distressed, and many labourers died. It was finally closed in 516, and was a great success.

- However, later it was not properly maintained.

"In the autumn, in the 8th month, the Huai River rose disastrously, the dam broke in many places, and the river flowed violently to the sea. Zu Geng was charged and sent to prison."

# WANG XIAOTONG 王孝通 (6TH–7TH CENT. CE)

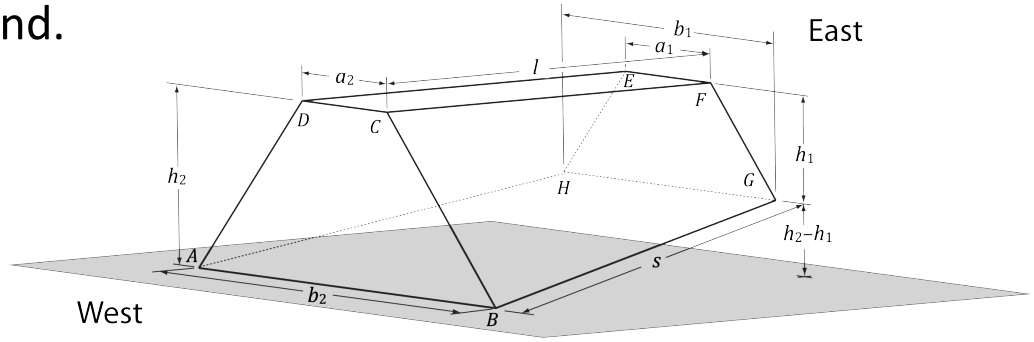
## Wang Xiaotong 王孝通 (6th–7th cent. CE)

- Served the Sui and Tang dynasties in posts concerned with calendrical calculations.
- Wrote *Jigu suanjing* 緝古算經, "Continuation of ancient mathematics".
  - One simple pursuit problem, stated as an astronomical problem.
  - 13 problems in solid geometry, stated as construction problems.
  - 6 problems in plane geometry, completely abstract.
- In 656 it was made one of the official "canons" for mathematical education.

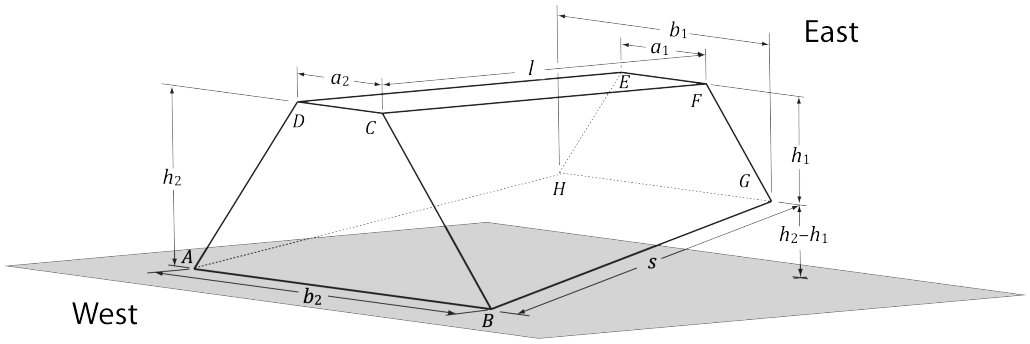
# *Jigu suanjing*, problem 3

(Unpublished work by Tina Su-lyn Lim)

Suppose a dyke is to be built. The difference between the upper and lower widths of the west end is 6 *zhang* 8 *chi* 2 *cun*, the difference between the upper and lower widths of the east end is 6 *chi* 2 *cun*, the height of the eastern end is 3 *zhang* 1 *chi* less than the height of the western end, the upper width is 4 *chi* 9 *cun* greater than the height of the eastern end, the straight length is 476 *chi* 9 *cun* greater than the height of the eastern end.




# *Jigu suanjing*, problem 3, continued




$b_2 - a_2 = 682$ *cun*
$b_1 - a_1 = 62$ *cun*
$h_2 - h_1 = 310$ *cun*
$a - h_1 = 49$ *cun*
$l - h_1 = 4769$ *cun*

$h_1^3 + 5004\, h_1^2 = 1{,}169{,}953\frac{1}{3}$ *cun*$^3$

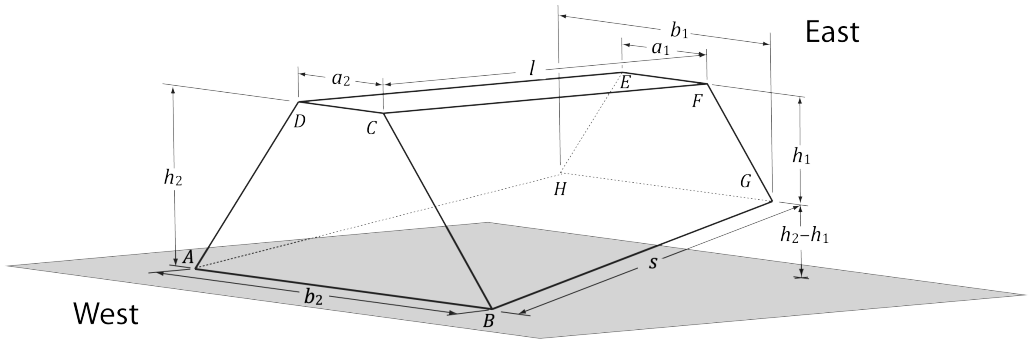
$h_1 = 41{,}107{,}188\frac{1}{3}$ *cun*

# *Jigu suanjing*, problem 3, continued

County A [sends] 6724 workers, county B 16677 workers, county C 19448 workers, county D 12781 workers. Each person from the four counties can during one day excavate 9 *dan* 9 *dou* 2 *sheng* [= 9920 *ge*] of soil. Each person can build a constant volume of 11 *chi* 4 $^{6}/_{13}$ *cun* [i.e. 114 $^{6}/_{13}$ *cun*$^3$ = 11 $^{29}/_{65}$ *chi*$^3$] per day. Digging out 1 [cubic] *chi* of soil results in 8 *dou* [= 800 *ge*] of soil.

People in former times, carrying 2 *dou* 4 *sheng* 8 *ge* [= 248 *ge*] of soil on their backs and travelling 192 *bu* on a level road, did 62 trips in one day. In the present situation there are hills to climb and rivers to cross to obtain the soil: there are only 11 *bu* of level road, the slanted height of the hill is 30 *bu*, and the width of the river is 12 *bu*. When climbing a hill 3 [*bu*] is equivalent to 4 [*bu* of even road], when descending a hill 6 [*bu*] is equivalent to 5 [*bu*], and when crossing water 1 [*bu*] is equivalent to 2 [*bu*]. • • •

# *Jigu suanjing*, problem 3, continued

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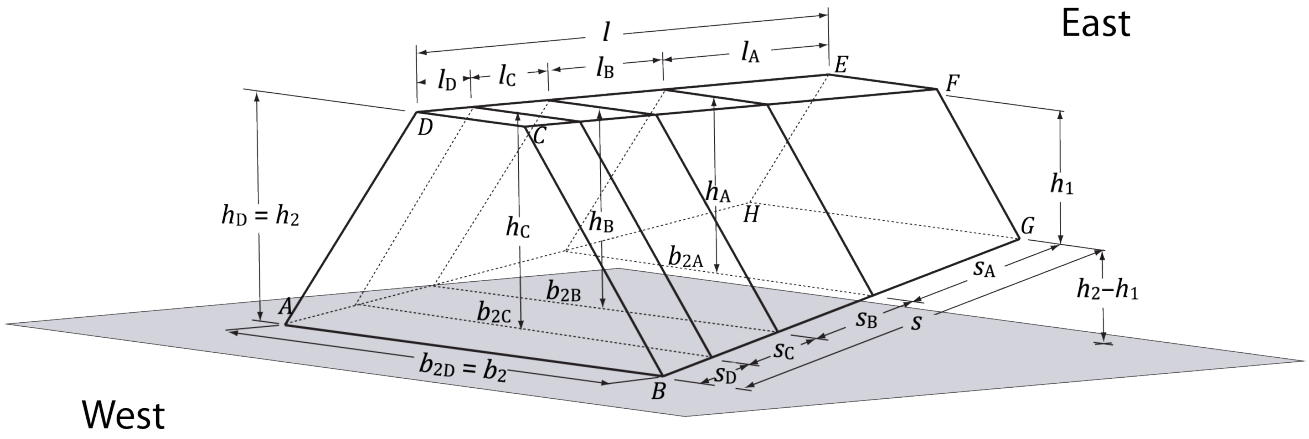
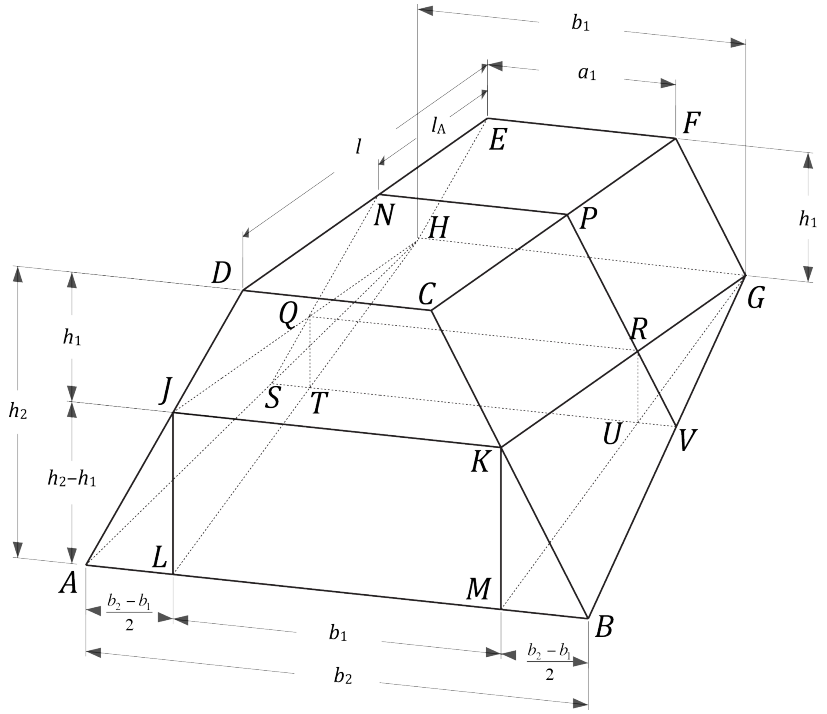
# *Jigu suanjing*, problem 3, continued




# *Jigu suanjing*, problem 3, continued

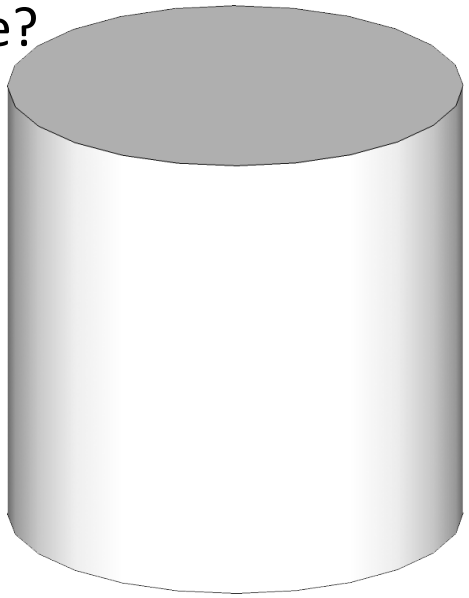


**TEXTBOOKS**

# Problem from a modern textbook for engineers

What relation between the height and the diameter of a tin can minimizes the amount of material used for a given volume?

Answer: *height* = *diameter*.



Pr[illegible]m a modern textbook

[illegible]ngineers

[illegible]ght a[illegible]the

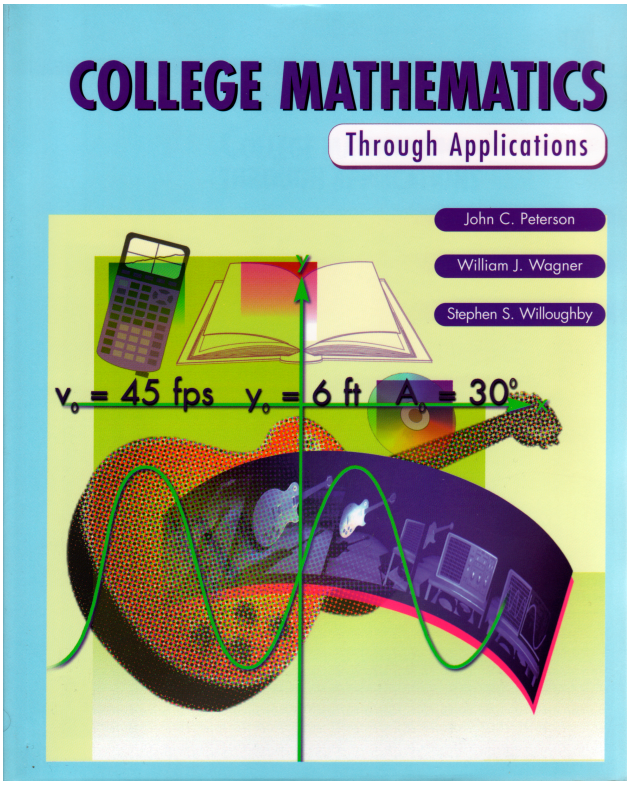
[illegible]n[illegible] the amount

[illegible]given volume?

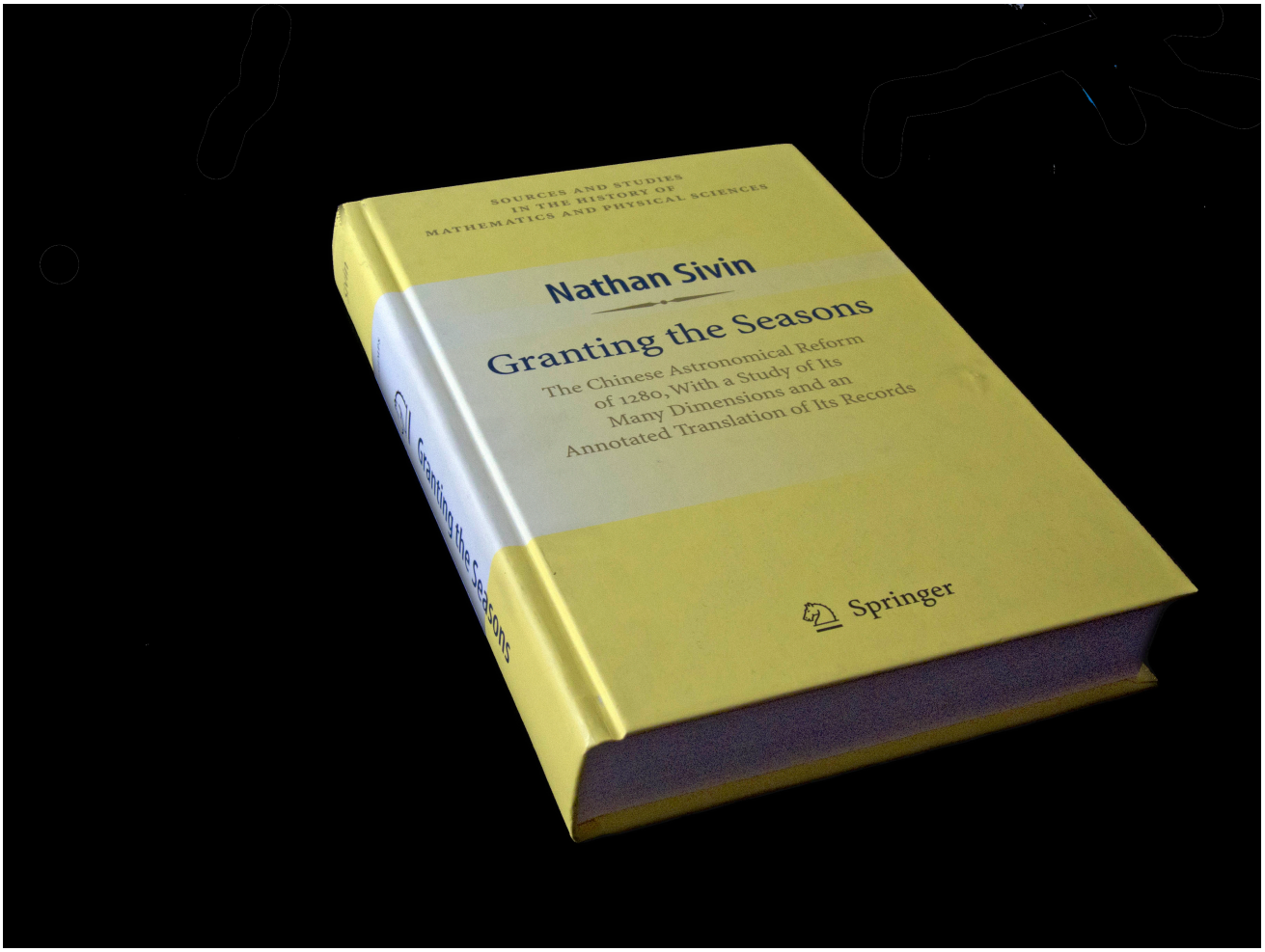
Answer: *neig*[illegible].


# A modern textbook for technical schools

http://www.amazon.com/s/field-keywords=0766802302



# GUO SHOUJING 郭守敬, 1231–1316



# Guo Shoujing 郭守敬, 1231–1316

(From the biography by Ho Peng Yoke in de Rachewiltz et al. *In the service of the Khan*, 1993, pp. 282–299.)

- His grandfather, Guo Rong, was well-educated in the classics and mathematics, and had an interest in water-works.
- In Xingzhou in 1251, when he was 20, the Pacification Commissioners, informed of Guo Shoujing's talents, had him produce plans and cost estimates for a bridge, and put him in charge • • •
- In 1262 Guo Shoujing presented six different schemes for improving irrigation and waterways in the region south of Yanjing and north of the Yellow River. • • • He was immediately appointed Superintendent of Waterways. The first of his suggestions was actually implemented.

# Guo Shoujing, continued

- In 1271 Guo Shoujing was made Director of Waterways.
- In 1275 he was ordered to survey the plain at the confluence of the Yellow and Huai Rivers with a view to building ports and waterways • • •.
- In 1291 he presented a scheme for improving transport on the Grand Canal • • •
- In 1298 he was summoned to plan a new canal • • •. He advised that the canal should be at least 77–107 metres wide to carry the summer surge. Senior officials, wishing to save costs, reduced the width by 1/3. The canal broke its banks in 1299, causing great loss of life and property.

## *HEFANG TONGYI* 河防通議

**PROTECTION WORKS ALONG THE YELLOW RIVER, EDITED BY SHAKESHI 沙克什, 1321.**

# *Hefang tongyi*, mathematical chapter, Problem 22.

Suppose there is a pit to be levelled with fascines. The length of the pit is 180 *bu*. The breadth is 20 *bu*, and the depth is 1 *zhang* 5 *chi*. For every 3 *chi* 6 *cun* [i.e., 3.6 cubic *chi*], one bundle of twigs and straw is used, using half of each . How much of each is used?

Answer: 1875 bundles.

Method: • • •

$$\frac{(180\ bu \times 5\ chi/bu) \times (20\ bu \times 5\ chi/bu) \times 15\ chi}{3.6\ chi^3\ /\ \text{bundle}} \times \frac{1}{2} = 1875\ \text{bundles}$$

# Fascines on a German canal

# Another fascine

**Into a Dike Breach Near Liaocheng Goes a Giant Bale of Kaoliang**

When it is in place, a foot of earth will be placed on top to make it settle deeper into the mud at the bottom of the Hwang Ho. This breach occurred during the disastrous flood of 1935, when 6,000 square miles of western Shantung farmland were inundated.

O. J. Todd, *National Geographic Magazine*, 1942, 81.2: 214.

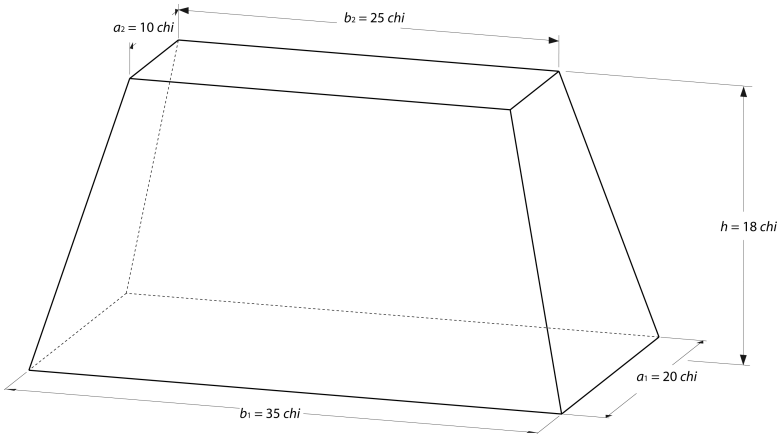
# *Hefang tongyi*, Problem 19

Suppose an ‘earthen ox’ [*tu niu* 土牛] is to be built. The upper length is 2 *zhang* 5 *chi*, the lower length is 3 *zhang* 5 *chi*, the upper breadth is 1 *zhang*, and the lower breadth is 2 *zhang*. The height is 1 *zhang* 8 *chi*. What is the volume?

Answer: 8100 [cubic] *chi*.

Method: • • •

# *Hefang tongyi*, Problem 19, continued




$$\left(\frac{25\ chi + 35\ chi}{2}\right) \times \left(\frac{10\ chi + 20\ chi}{2}\right) \times 18\ chi = 8100\ chi^3$$

Correct calculation:

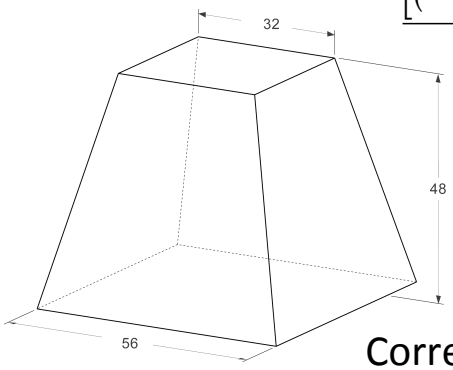
$$\frac{\left(2 \times 25\ chi + 35\ chi\right) \times 10\ chi + \left(2 \times 35\ chi + 25\ chi\right)}{6} \times 18\ chi = 8250\ chi^3$$

# *Hefang tongyi*, Problem 18

Suppose a square terrace is to be built. Above it is 3 *zhang* 2 *chi* square, below it is 5 *zhang* 6 *chi* square, and the height is 48 *chi*. What is the volume?

Answer: 7936 [cubic] *chi*.

Method: • • •




$$\frac{\left[(32\ chi)^2 + (56\ chi)^2 + (32\ chi \times 56\ chi)\right] \times 48\ chi}{36} = 7936\ chi^3$$

Correct calculation:

$$\frac{\left[(32\ chi)^2 + (56\ chi)^2 + (32\ chi \times 56\ chi)\right] \times 48\ chi}{3} = 95{,}232\ chi^3$$